Intelligence by the Numbers

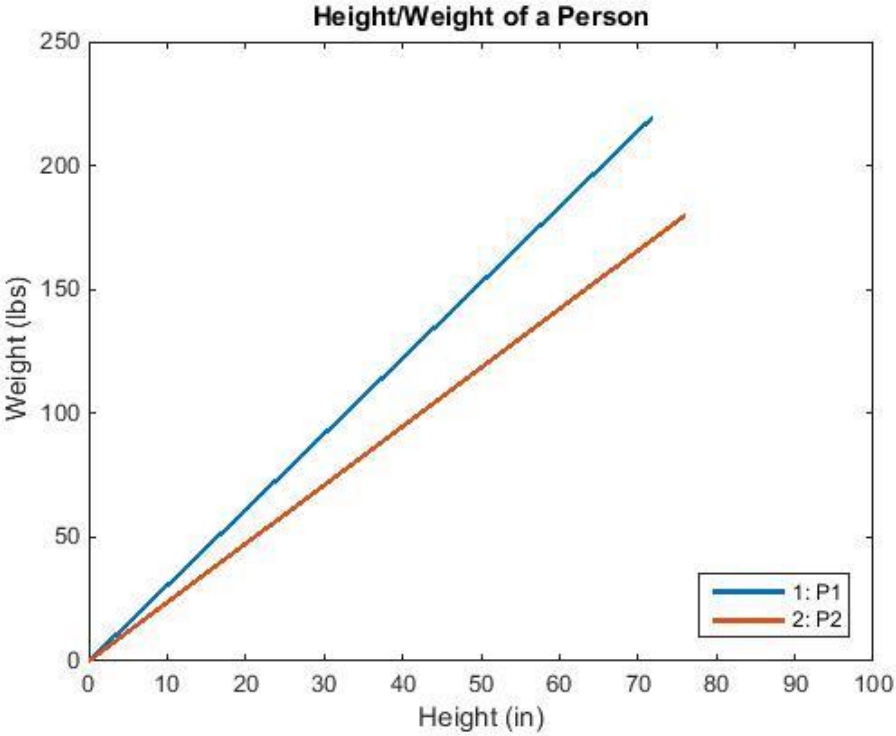
In this chapter I lay out the first part of mathematical foundation for comparing the capabilities of machines to humans. Let me begin by saying that this is not intended to be a mathematical proof that machines can be more intelligent than humans and I have made no attempt at approaching this with any sort of mathematical rigor. The purpose of this exposition is simply to focus attention on existing mathematical concepts in the context of the ongoing discussion regarding the metrics we use to measure intelligence. I will later use this to show why I believe that machines will far outperform us no matter what criteria we might adopt in measuring intelligence. The notion of intelligence and intelligent behavior is an abstract one and any attempts to quantify intelligence in humans, animals, or other entities, both living and non-living, have fallen short of expectations. Mathematics has longed served as a powerful tool for abstraction of concepts and manipulating those concepts in useful ways. We have successfully deployed it as a tool for working in domains where we have a solid understanding, such as chemistry, as well as domains in which we are still discovering their true nature, such as quantum field theory. Perhaps the most famous example of using mathematic to represent concepts which were barely understood is Einstein's theories of relativity. In this case we see an example of using mathematics to build a theory which predicts the existence of phenomenon which were formerly unknown. By representing the world around us in abstract, i.e. mathematical terms, we are able to use the rich set of mathematical tools at our disposal to test hypotheses and follow them to their logical conclusion.....

Representing Qualities with Vectors

Begin by considering the mathematical concept of the vector, which we see used in many domains throughout business and science alike . The notion of a vector is a simple one. It is a series of numbers which comprise a collection treated as a single unit. Although vectors can be manipulated and compared completely independently of any association with real world values, for our purposes we will assume that they represent something 'real'. Choosing a simple example will make it easier to introduce some concepts typical of vectors . For example, we might use a vector to represent a person with respect to two values such as height and weight. Most of us are familiar with graphing values on an X/Y axis so we will use this to review some simple concepts before moving on to more useful examples. Let's start with a vector to represent a person (P_1) with a height of 72 inches and weight of 220 pounds and another person(P_2) with a height of 76 inches and weight of 180 pounds, represented as:

$P_1 = [72, 220]$ $P_2 = [76, 180]$

These two vectors represent our somewhat simplistic categorization as depicted in 2-dimensional space below.



We can easily see that P_1 is greater in terms of weight and that P_2 is greater in height. We can also see that taking the two attributes combined (if we assume that height and weight are equally important) P_1 is slightly greater than P_2 because the line representing P_1 is longer than the line representing P_2 . This measurement is known as the magnitude of a vector. We say that the magnitude of P_1 is greater than the magnitude of P_2 .

Multiplying a vector by some value x changes the length of the vector by that value x. These are known as scalars. For our examples we will limit our scalars to integers. If we multiply v_1 by 2 it becomes *{144, 400}* or graphically FIGURE

Now consider a second vector, $v_2 = \{60, 150\}$ which represents a different person with a height of 60 inches and weight of 150 pounds.

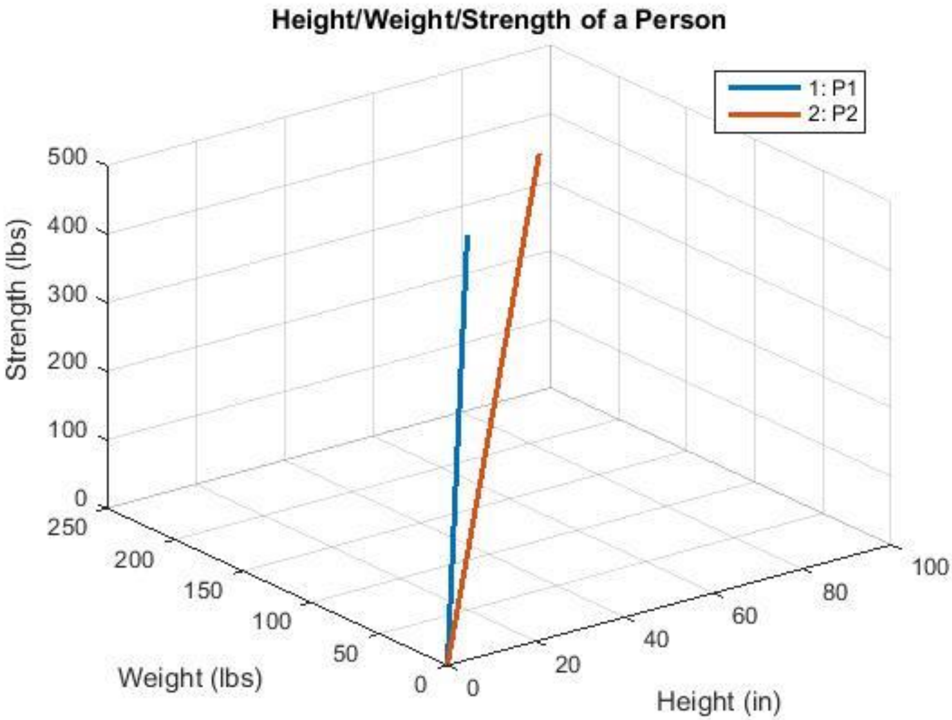
We can represent these two vectors graphically as FIGURE

Now let's consider a simple operation such as addition. We can represent $v_1 + v_2 = \{72+60, 200+150\} = \{132, 350\}$

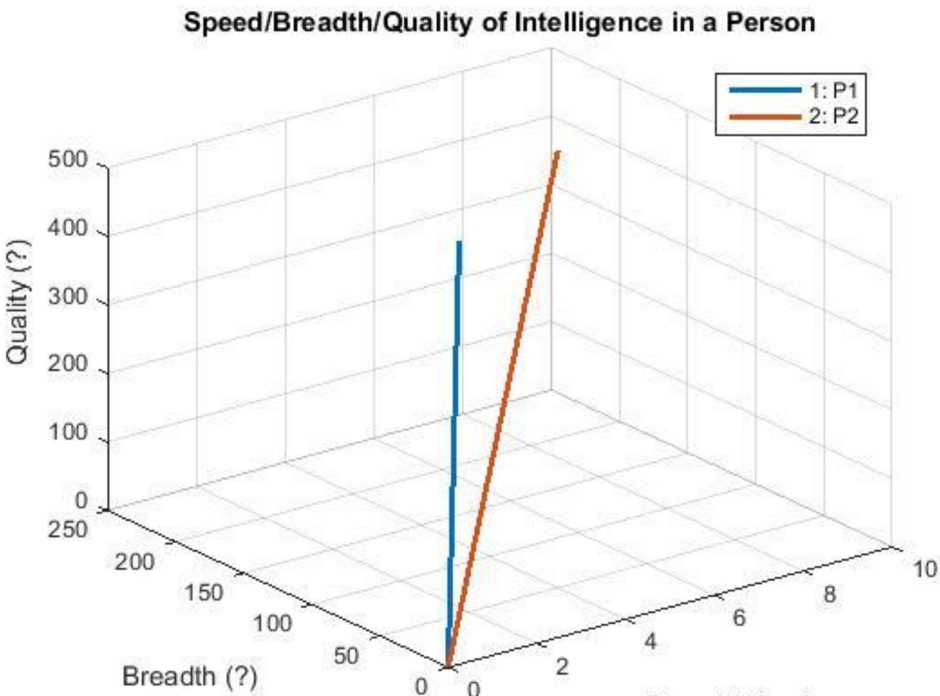
or graphically FIGURE

It is important to understand what this means beyond simply adding numbers together and drawing graphs against an X/Y axes. What we are saying is that if we take one person who weighs 200 pounds and another who weighs 150, together they will weigh 350 pounds. While this may seem painfully obvious it is important to keep this in mind before we move to more complex examples. And what about height? While it may not be typical to stack one person on top of another, if were to do so these two well-balanced test subjects would have a total height of 132 inches. (We might find it easier to lay them end to end.)

There is an important concept in vector math known as magnitude. This is a way of measuring the size of the vector as a whole. If we compare two vectors with values {80, 100} and {100,80} and ask which is larger we might say that the first is smaller in terms of its first component but larger in terms of its second component. But the two components combined have an overall magnitude of just over 141 which is arrived at by taking the square root of the sum of the squares of the components. And of course the magnitude of the second vector is also just over 141. So although the two vectors are different and each one is larger the other relative to one component, they are of equal magnitude. We might relate this measure as a general measure of a person's size which considers both their height and weight.



Height/Weight/Strength of a Person
1: P1
2: P2
Strength (lbs)
500
400
300
200
100
0
250
200
150
100
50
0
Weight (lbs)
0
20
40
60
80
100
Height (in)


Speed/Breadth/Quality of Intelligence in a Person
1: P1
2: P2
Quality (?)
500
400
300
200
100
0
250
200
150
100
50
0
Breadth (?)
0
2
4
6
8
10
Speed (1/sec)

As